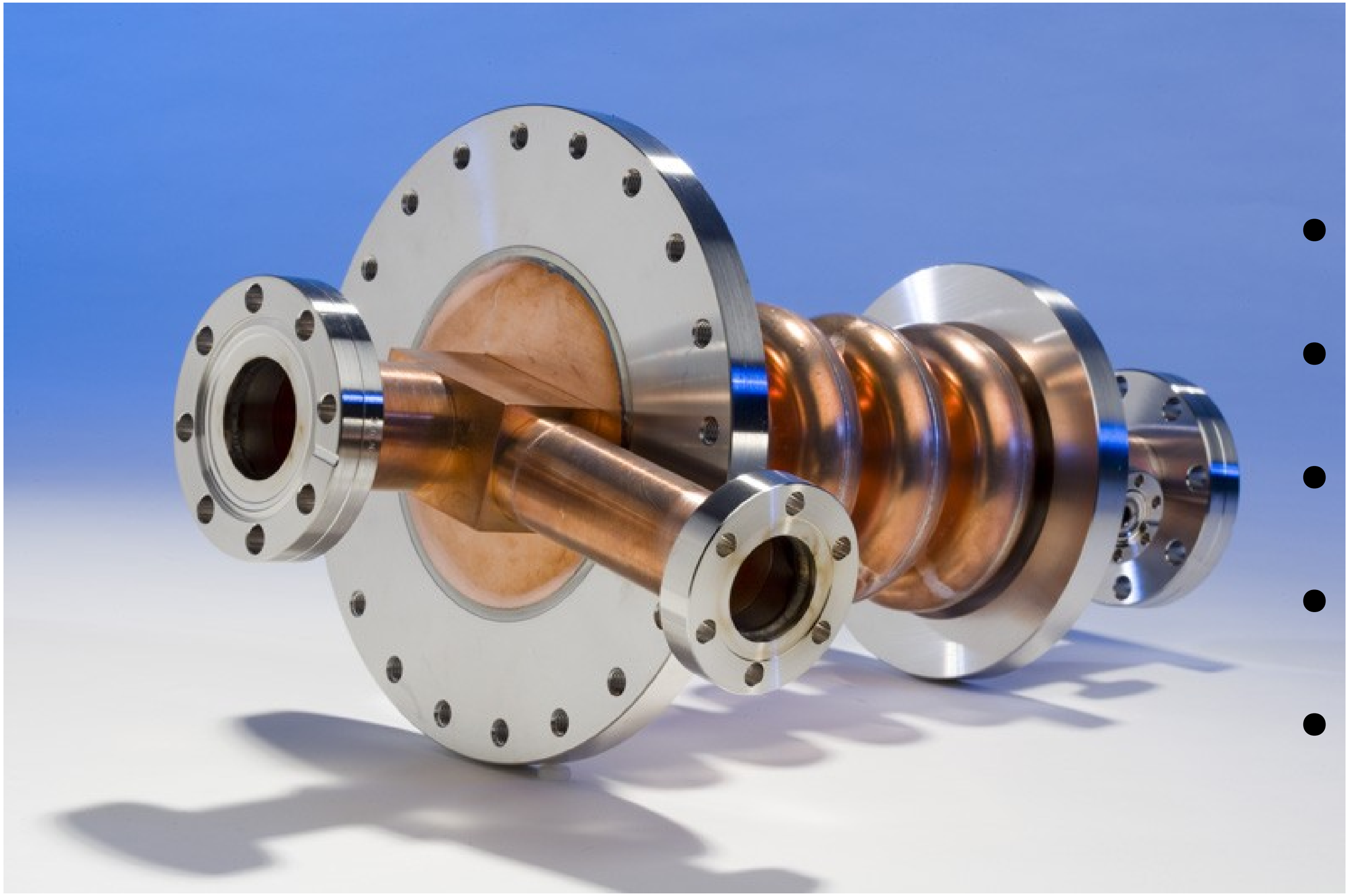
Determination of the cavity matrix

Emmanuel Branlard

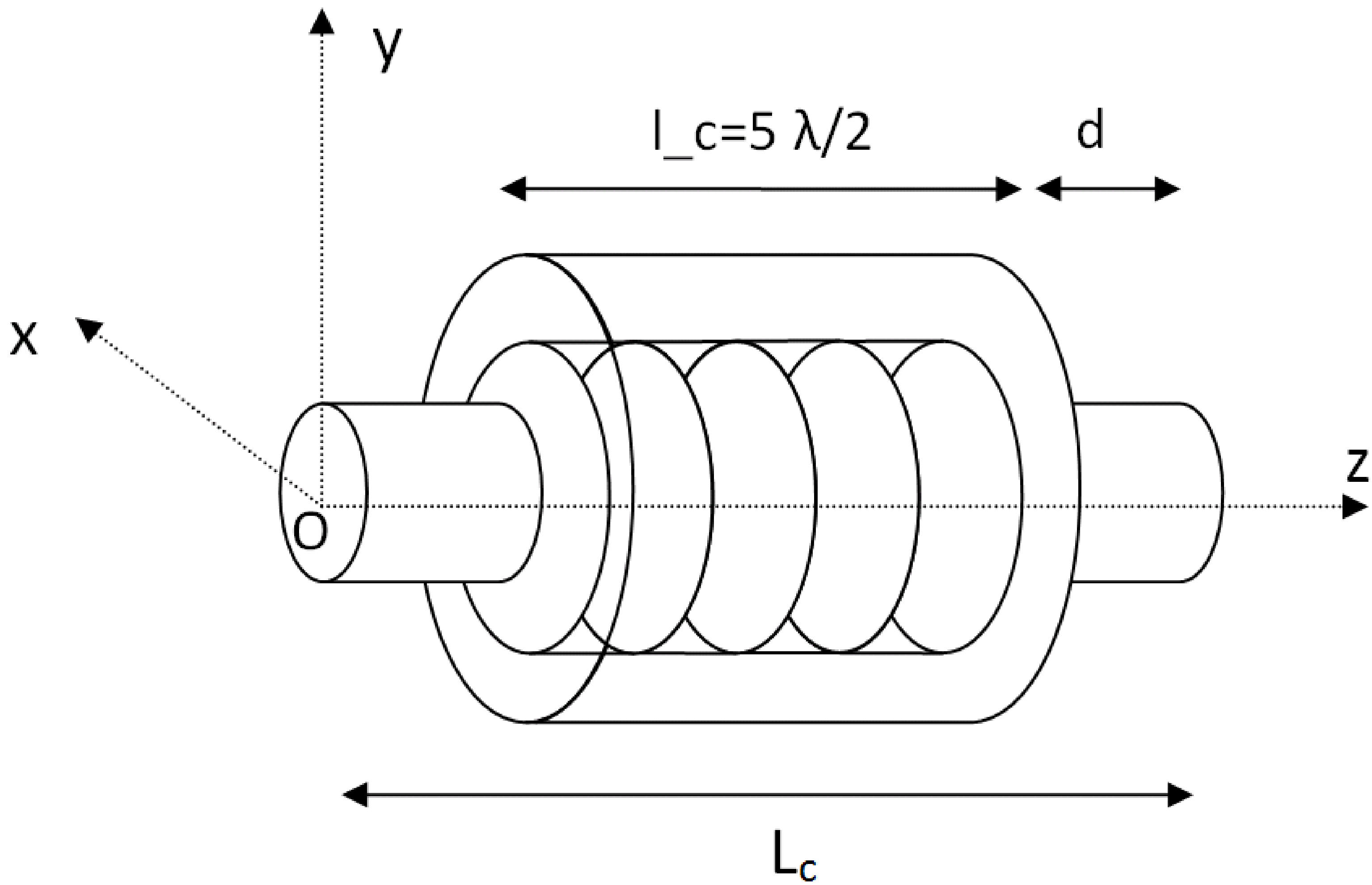
- Presentation of the cavity
- Matrix formalism
- The pill box approximation
- Numerical matrix determination
- Plots and final comparisons

Presentation of the cavity



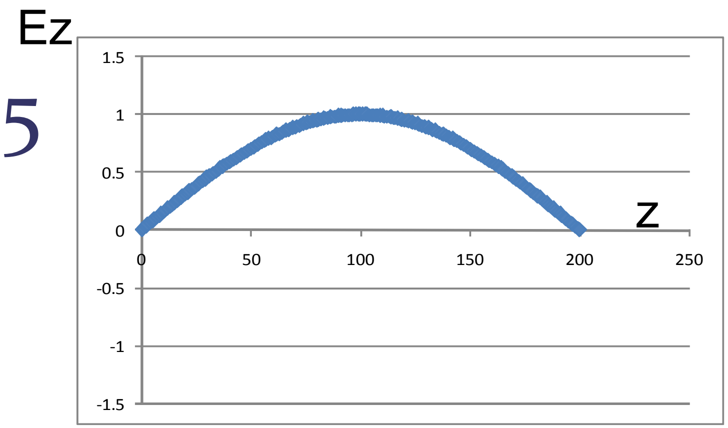
- Copper cavity
- 3.9 Ghz
- 5 cells
- TM-110 mode
- Pi mode

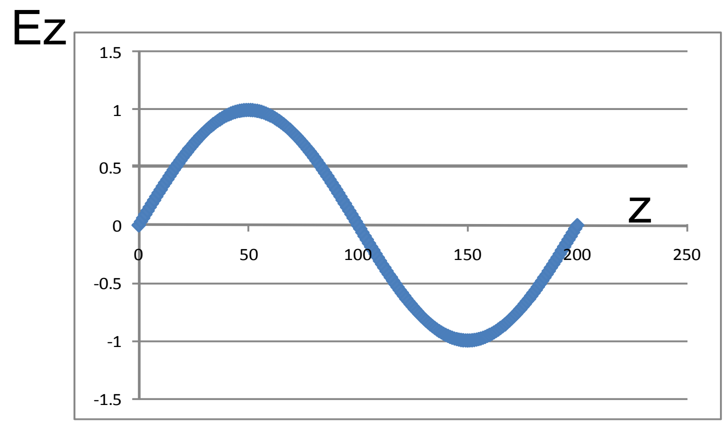
Usual notations



The pi mode

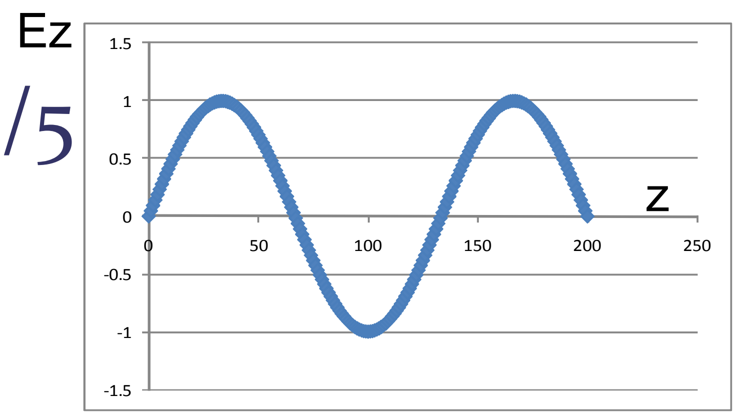
- pi/5

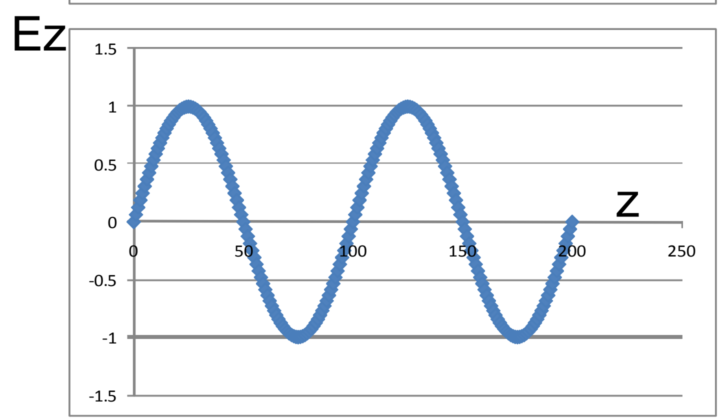




- 2pi/5

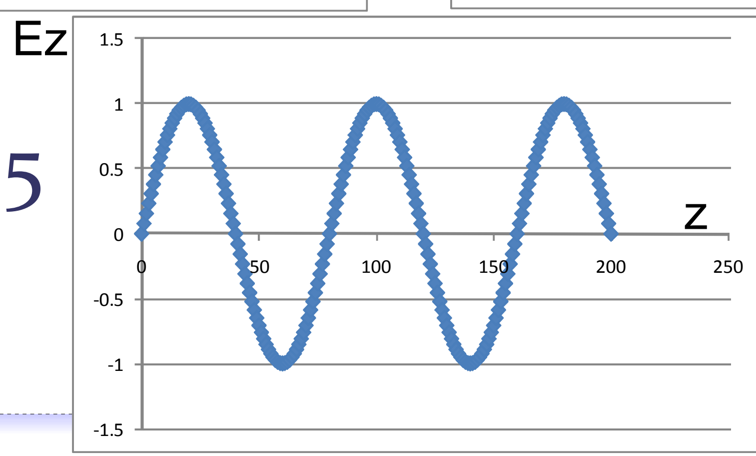
- 3pi/5





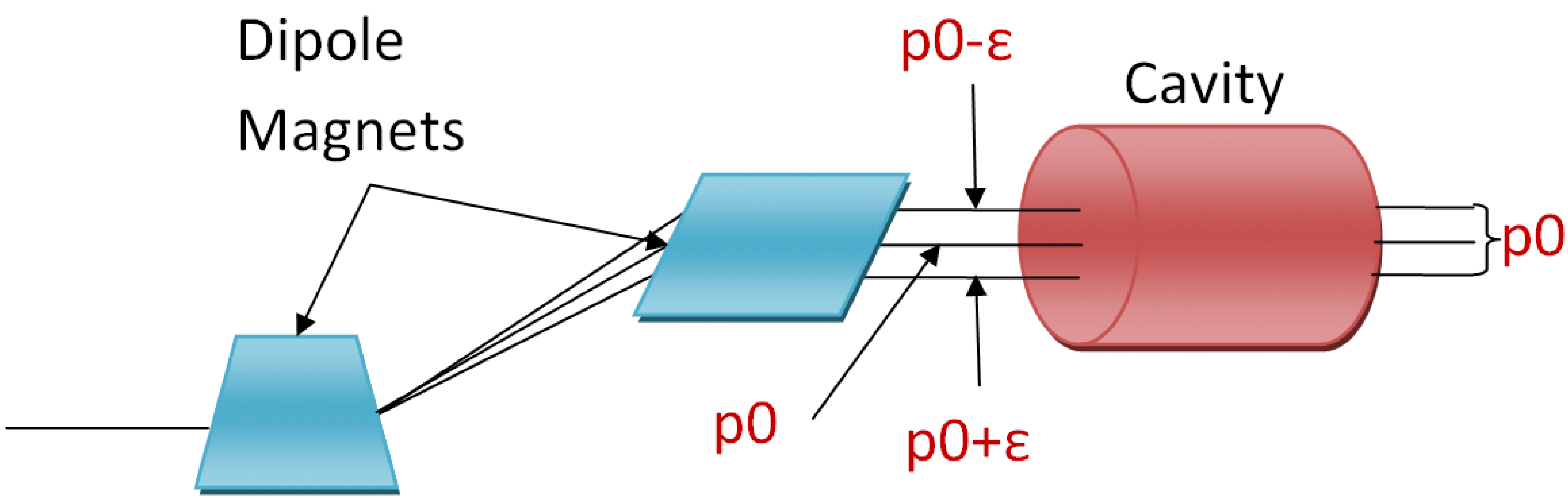
- 4pi/5

- 5pi/5



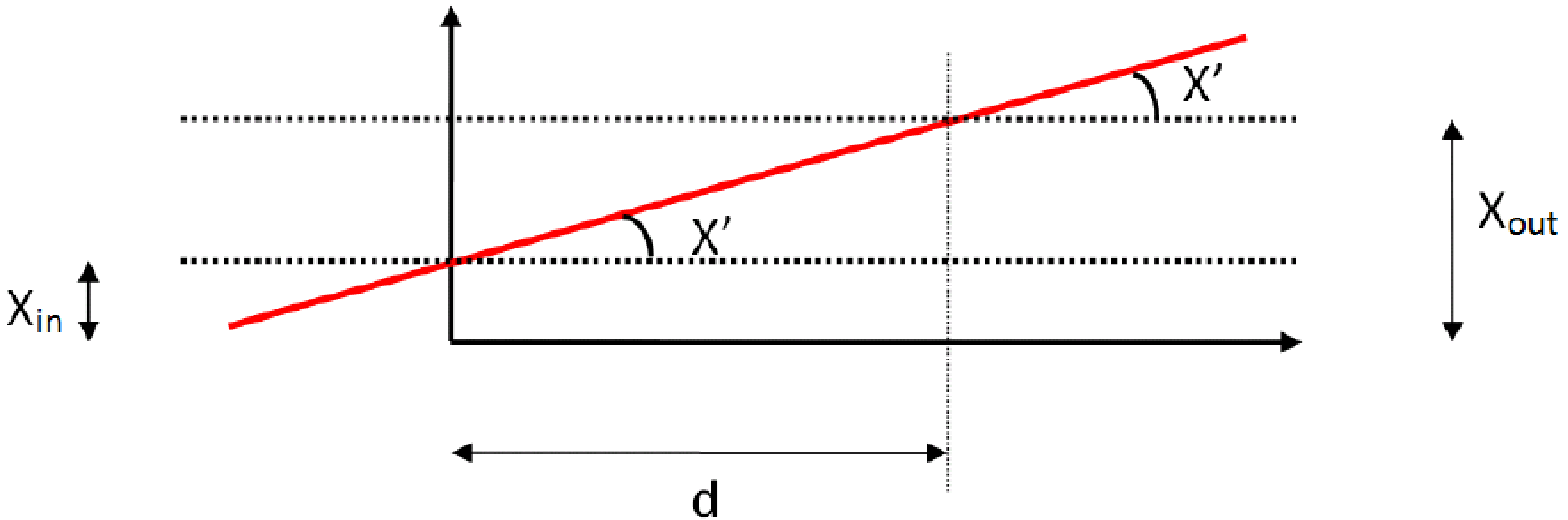
The aim of the cavity

- Exchange transverse and longitudinal emittance



Matrix formalism

- For a simple drift



$$\begin{pmatrix} x_{out} \\ x'_{out} \end{pmatrix} = \begin{pmatrix} x_{in} + tan(x')d \approx x + d.x' \\ x'_{in} \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ x'_{in} \end{pmatrix}$$

Problem definition

$$V = \begin{pmatrix} x & (m) \\ x' & (rad) \\ y & (m) \\ y' & (rad) \\ z & (m) \\ \frac{\delta p}{p} & (\oslash) \end{pmatrix}$$

$$V_{out} = A.V_{in}$$

-> Finding the cavity matrix A

The pill box approximation with the work of Don Edwards

- The fields

$$E_s(X,t) = E' X \cos(\omega t)$$
$$B_y(X,t) = \frac{E'}{\omega} \cos(\omega t)$$

- Equation of motion :

$$\frac{dP_{X,r}}{ds} = \frac{-eE'}{kc} \sin(ks)$$
$$\frac{dP_{S,r}}{ds} = \frac{eE'}{c} X_r \cos(ks) + \frac{eE'}{kc^2} V_{X_r} \sin(ks)$$

- Doing the same for the reference particle, after integration we have :

$$x'(s) = x'_{in} + \frac{Tp}{2} z\, sin(ks) + \frac{Tp}{2} z$$

$$x(s) = x_{in} + \left(x'_{in} + \frac{Tp}{2} z\right)\left(s + \frac{\lambda}{4}\right) - \frac{Tp}{2k} z\, cos(ks)$$

$$\begin{aligned} \frac{\delta p}{p} &= x_{in}\, \frac{T}{2} \left[1 + sin(ks)\right] \\ &+ x'_{in}\, \frac{T}{2} \left[\left(s + \frac{\lambda}{4}\right) sin(ks)\right] \\ &+ z\, \frac{T^2}{4} \left[\left(s + \frac{\lambda}{4}\right) sin(ks) - \frac{1}{2k} sin(2ks)\right] \end{aligned}$$

Matrix in the pill box approximation

$$M_{cell} = \begin{pmatrix} 1 & \lambda/2 & T\lambda/4 & 0 \\ 0 & 1 & T & 0 \\ 0 & 0 & 1 & 0 \\ T & T\lambda/4 & T^2\lambda/8 & 1 \end{pmatrix}$$

$$M_{cav} = M_d . M_{cell}^5 . M_d$$

$$M_{cav} = \begin{pmatrix} 1 & 2d + 5\lambda/2 & 5Td + 25T\lambda/4 & 0 \\ 0 & 1 & 5T & 0 \\ 0 & 0 & 0 & 0 \\ 5T & 5Td + 25T\lambda/4 & 85T^2\lambda/8 & 1 \end{pmatrix}$$

Numerical matrix determination

- The method

$$\begin{pmatrix} A_{11} & . & A_{1n} \\ . & . & . \\ A_{i1} & . & A_{in} \\ . & . & . \\ . & . & . \\ A_{n1} & . & A_{nn} \end{pmatrix} . \begin{pmatrix} 0 \\ . \\ \lambda \\ . \\ . \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} A_{1i} \\ . \\ A_{ii} \\ . \\ . \\ A_{ni} \end{pmatrix}$$

- The input vectors

$$V_0 = \vec{0} \; V_1 = \begin{pmatrix} 0.001 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} V_2 = \begin{pmatrix} 0 \\ 0.01 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} V_3 = \begin{pmatrix} 0 \\ 0 \\ 0.001 \\ 0 \\ 0 \\ 0 \end{pmatrix} V_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.01 \\ 0 \\ 0 \end{pmatrix} V_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.001 \\ 0 \end{pmatrix} V_6 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.1 \end{pmatrix}$$

The RF phase

- Ez field

$$E_z(z,t) = E_{z0}(z).sin(\omega t + \phi)$$

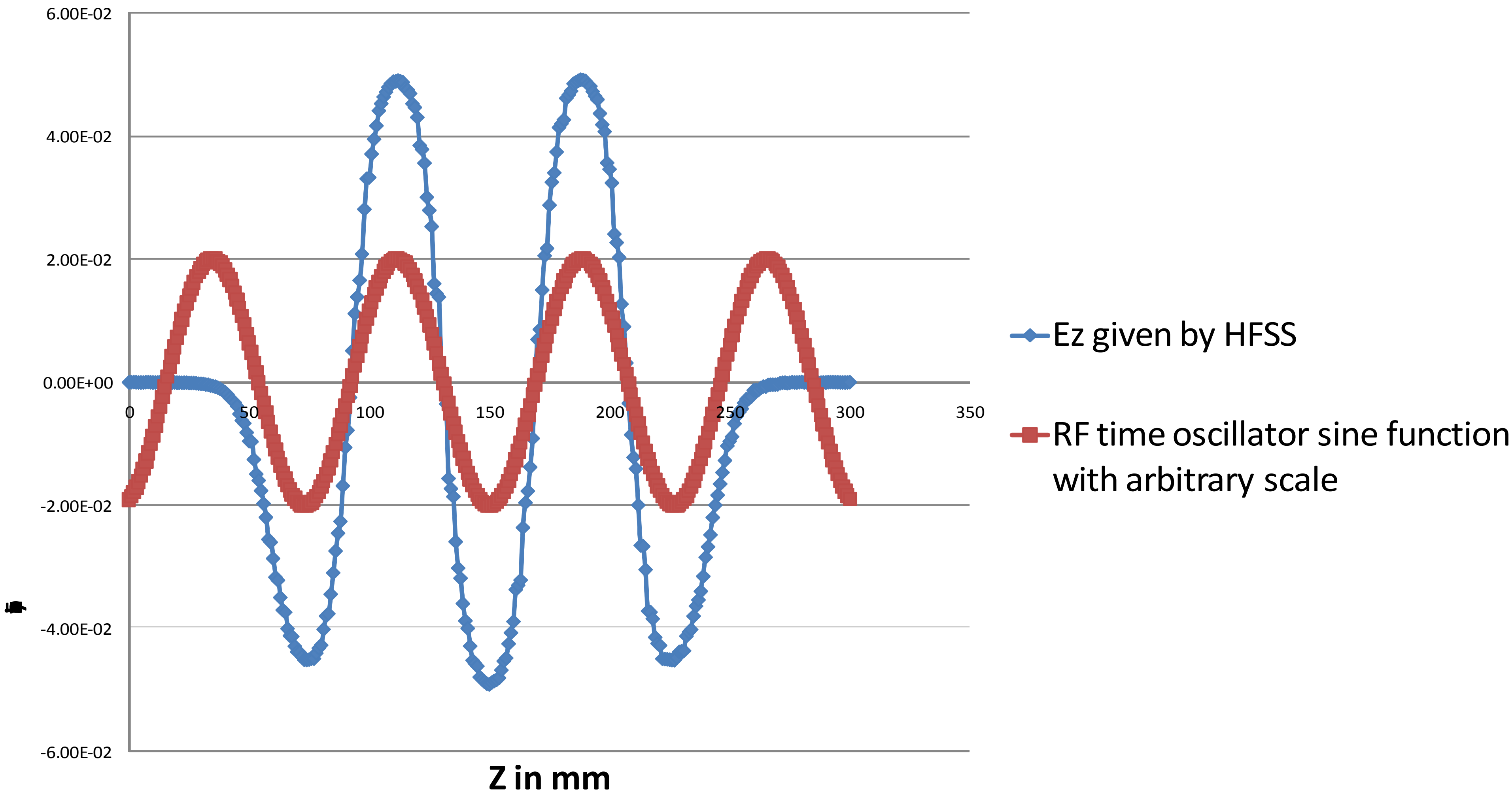
- For a particle at speed of light : z(t)=ct

$$E_z^p(z) = E_z(z, \frac{z}{c}) = E_{z0}(z).sin(2\pi\frac{z}{\lambda} + \phi)$$

$$2\pi\frac{L_c}{2\lambda} + \phi = \frac{-\pi}{2}$$

$$\phi = 287.58 \text{ deg}$$

The Ez field seen by a "particle" travelling on the axis x=1mm, split in a product of two functions : a sine function and a spatial function

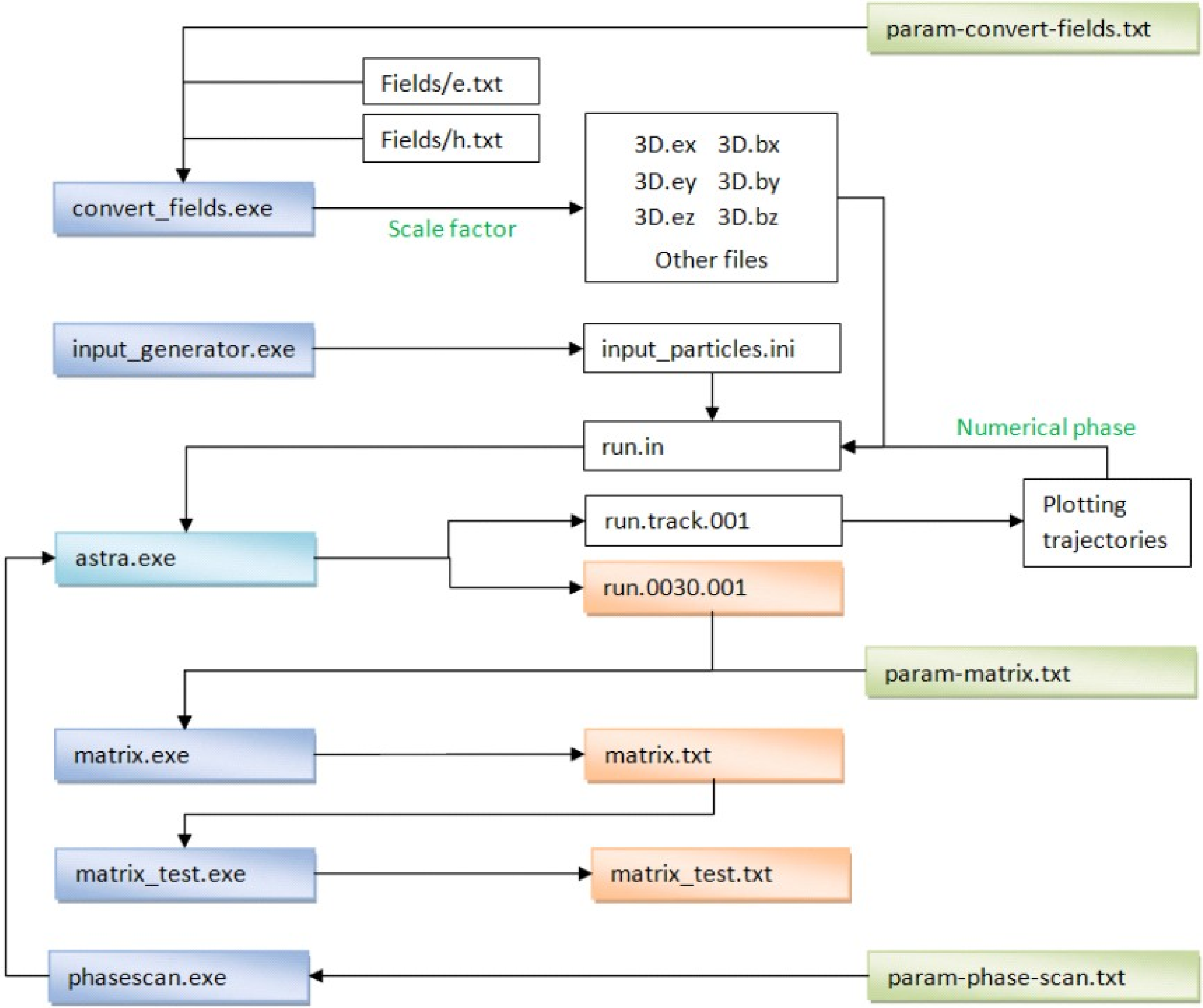


The scaling factor

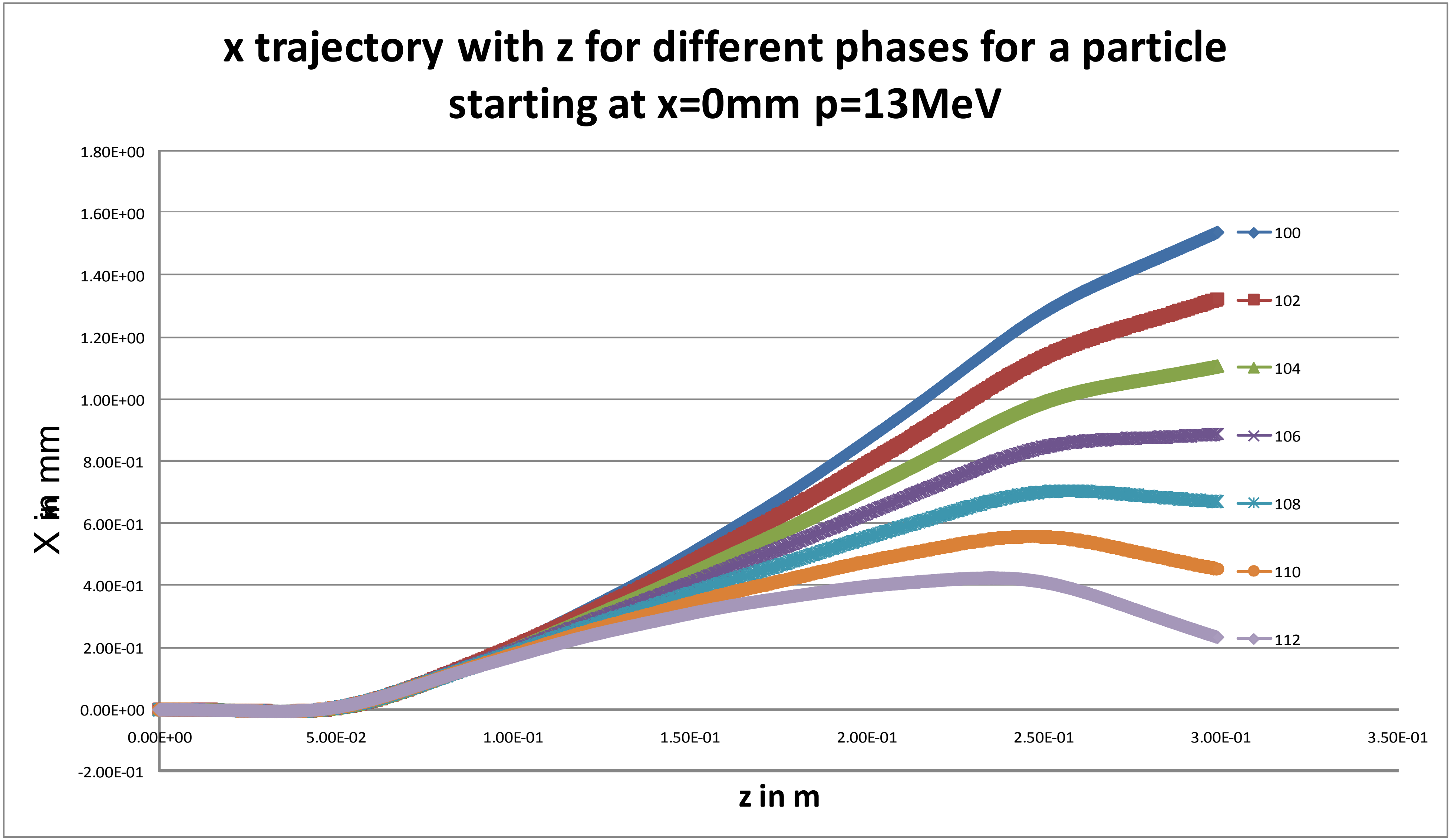
$$V = \int_0^{L_c} E_z^p.dz$$

$$V = \sum_i \int_{z_i}^{z_{i+1}} E_z^p(z).dz = \sum_i E_z^p(z_i) \int_{z_i}^{z_{i+1}} dz = \sum_i E_z^p(z_i).z_{step}$$

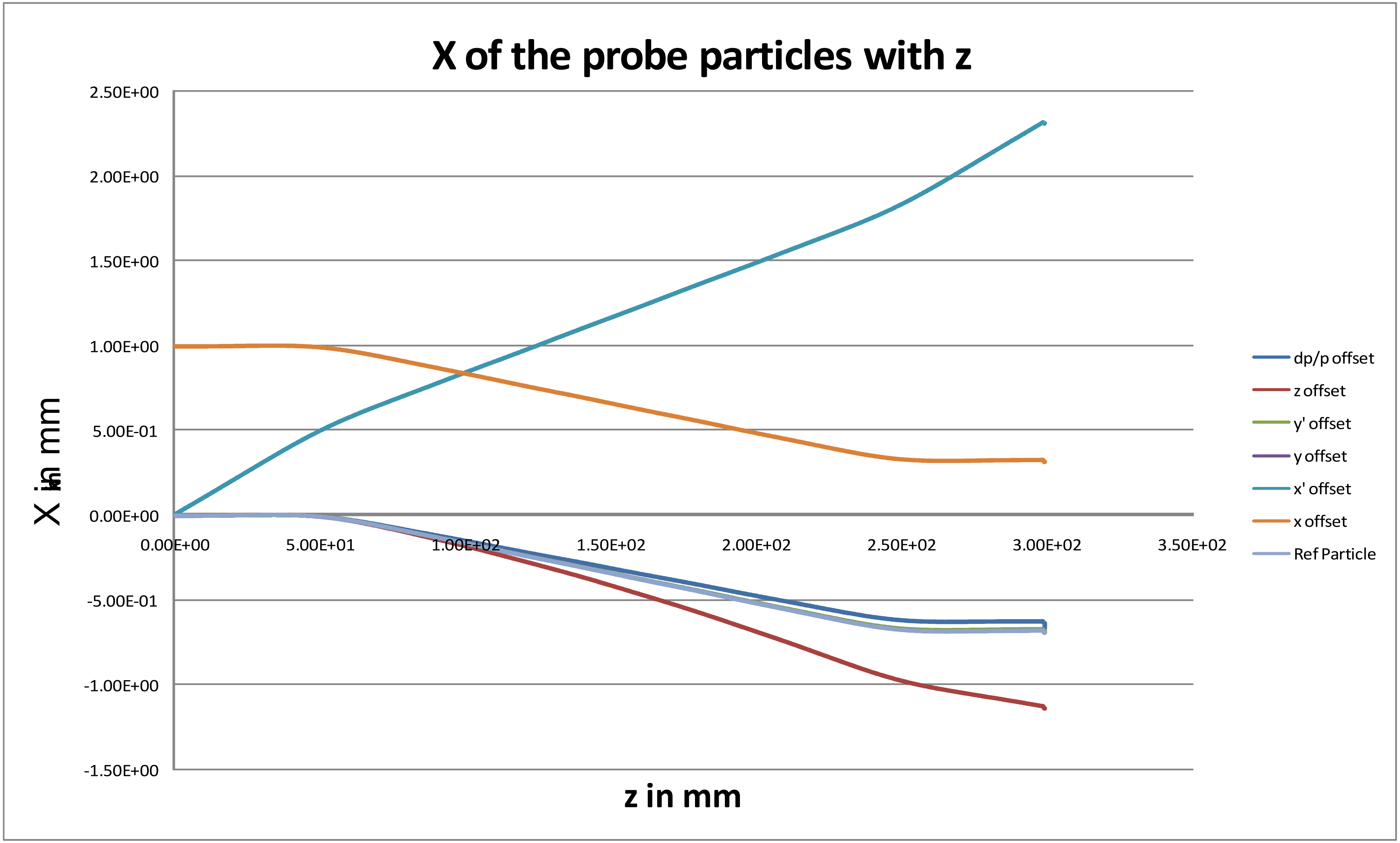
$$V = 9.3227\text{e}+6 \ \text{V/m}$$


param-convert-fields.txt
Fields/e.txt
Fields/h.txt
convert_fields.exe
Scale factor
3D.ex 3D.bx
3D.ey 3D.by
3D.ez 3D.bz
Other files
input_generator.exe
input_particles.ini
run.in
Numerical phase
Plotting trajectories
astra.exe
run.track.001
run.0030.001
param-matrix.txt
matrix.exe
matrix.txt
matrix_test.exe
matrix_test.txt
phasescan.exe
param-phase-scan.txt

Plots and final comparisons



X deflection

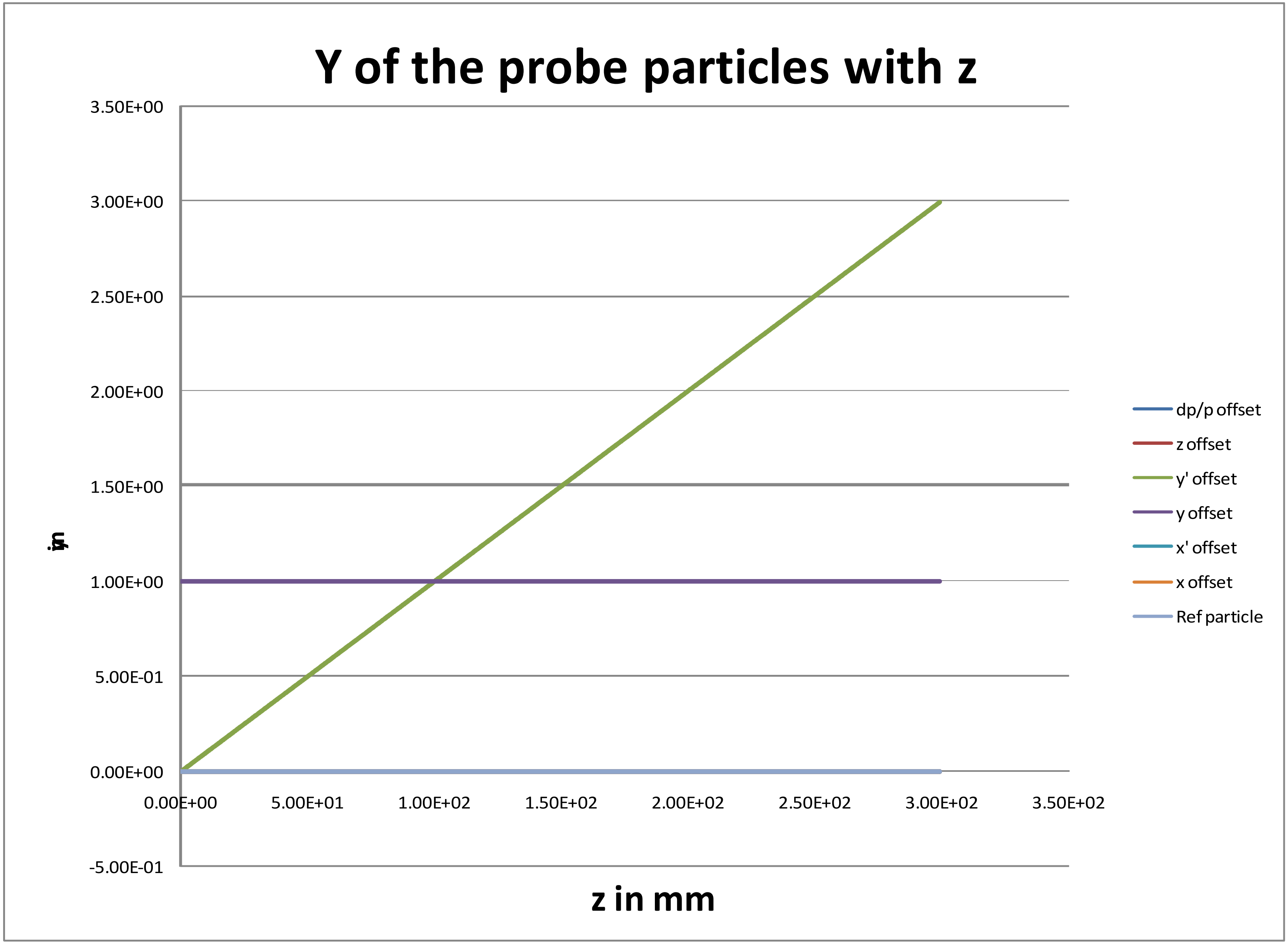


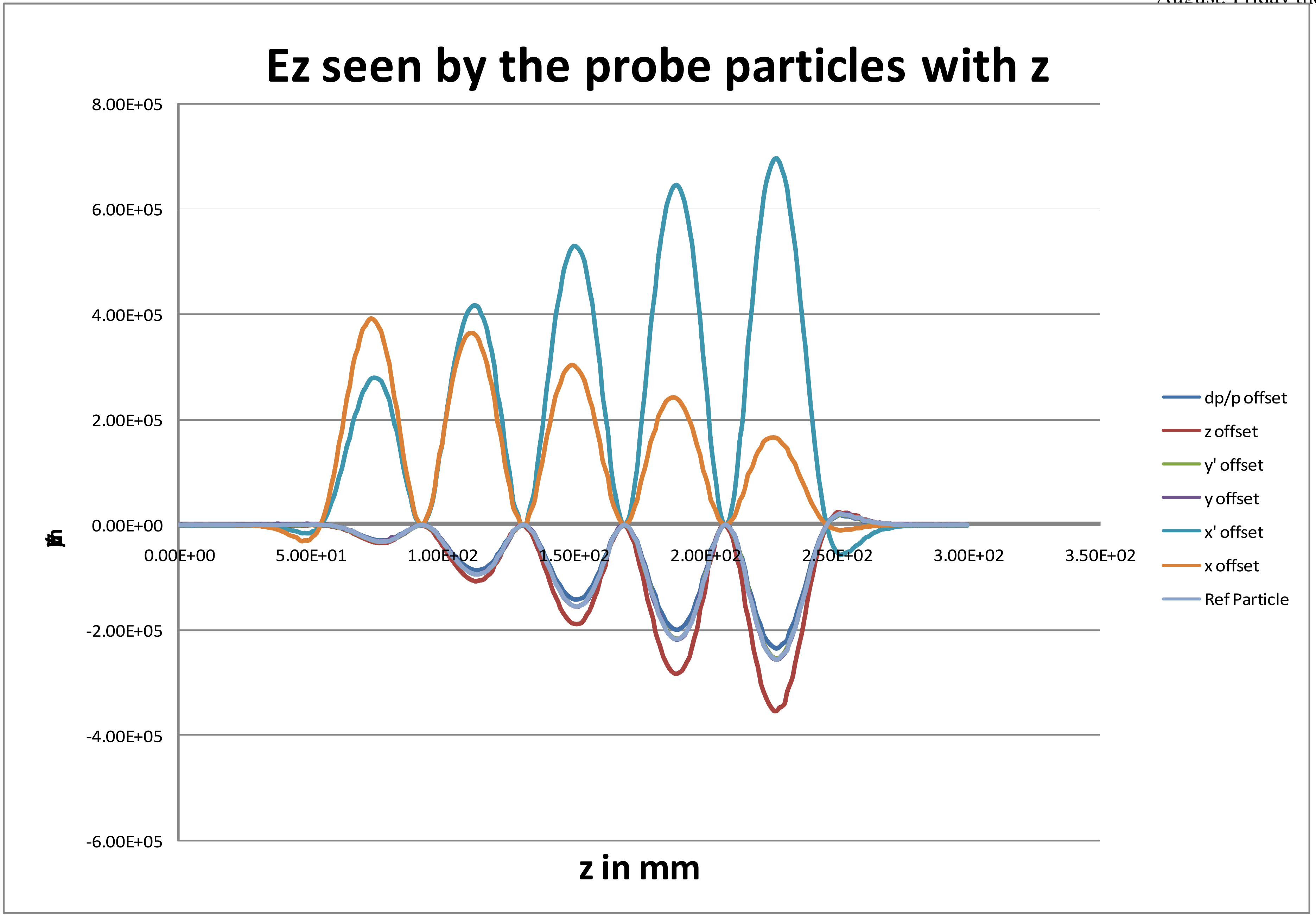
The final matrix

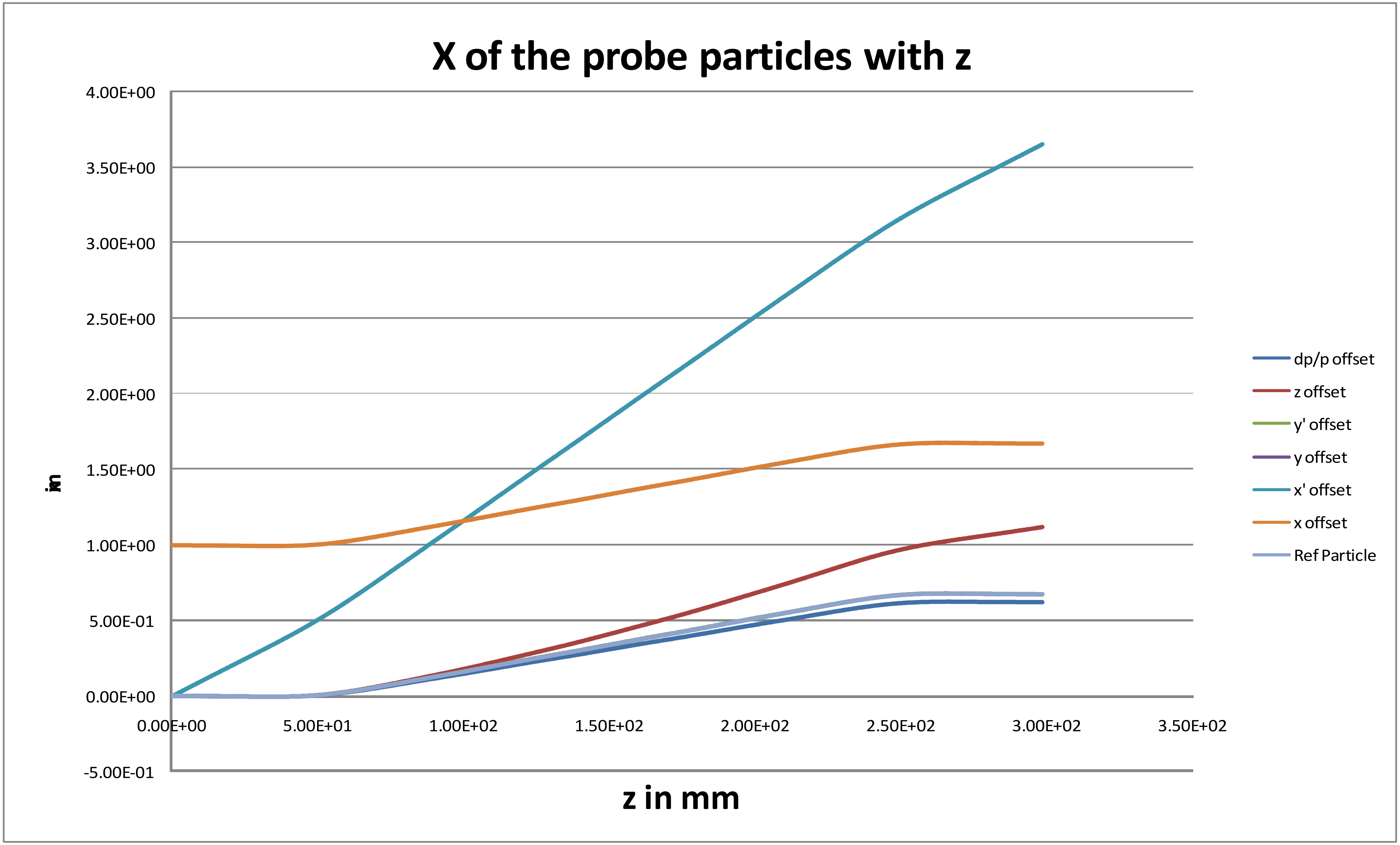
$$M_{cav} = \begin{pmatrix} 1.003 & 0.300 & 0.002 & 0.001 & -0.448 & 0.001 \\ 0.020 & 1.004 & -0.002 & 0.002 & -2.991 & -0.000 \\ 0.000 & 0.000 & 1.002 & 0.300 & -0.000 & 0.000 \\ 0.002 & -0.000 & 0.012 & 1.002 & -0.000 & 0.000 \\ -0.001 & -0.001 & 0.000 & -0.001 & 0.998 & 0.000 \\ -3.001 & -0.450 & -0.001 & -0.001 & 0.275 & 0.999 \end{pmatrix}$$

$$M_{cav} = \begin{pmatrix} 1 & 0.300 & 0 & 0 & -0.452 & 0 \\ 0 & 1 & 0 & 0 & -3.016 & 0 \\ 0 & 0 & 1 & 0.300 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -3.016 & -0.452 & 0 & 0 & 0.297 & 1 \end{pmatrix}$$

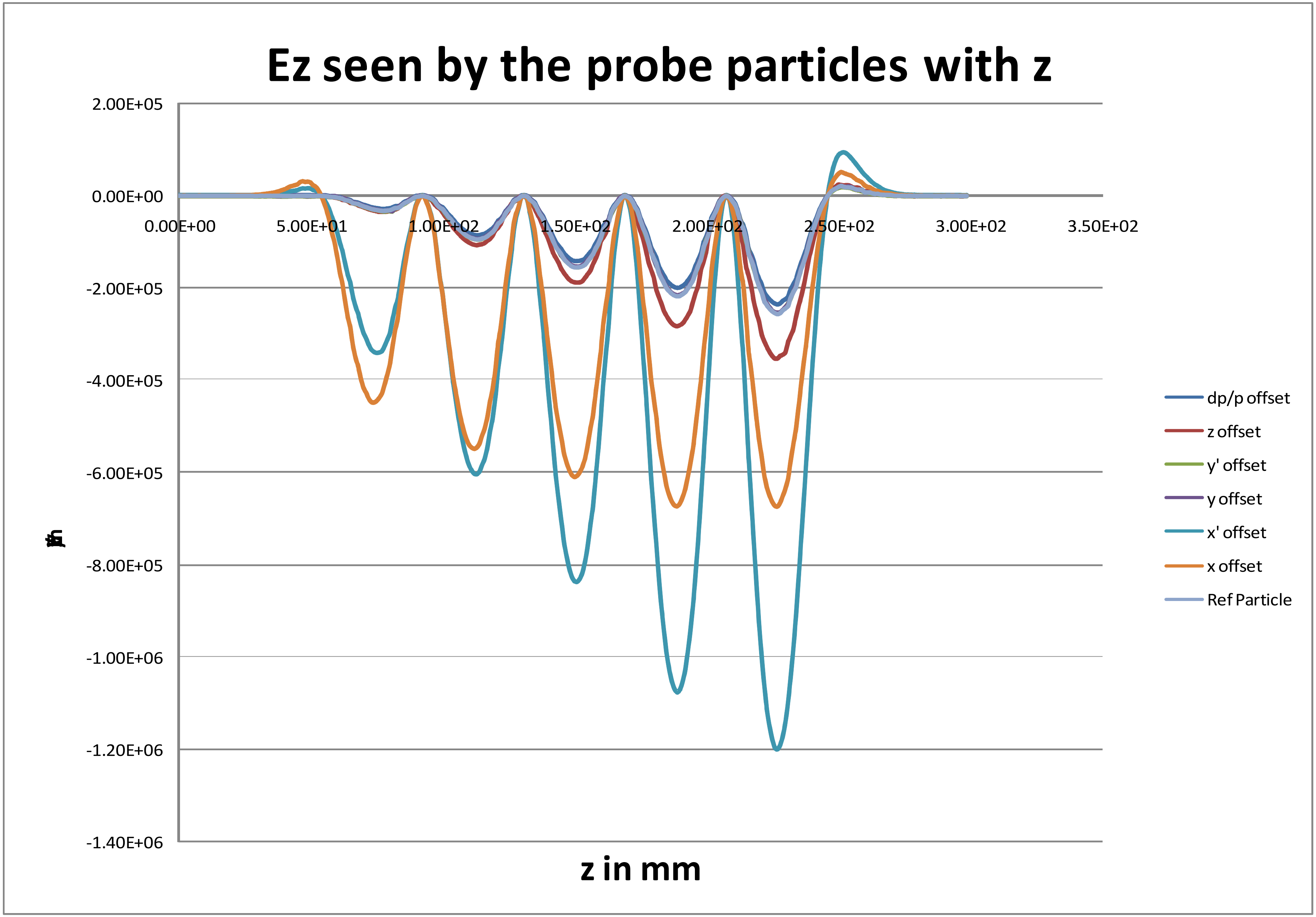
Y deflection

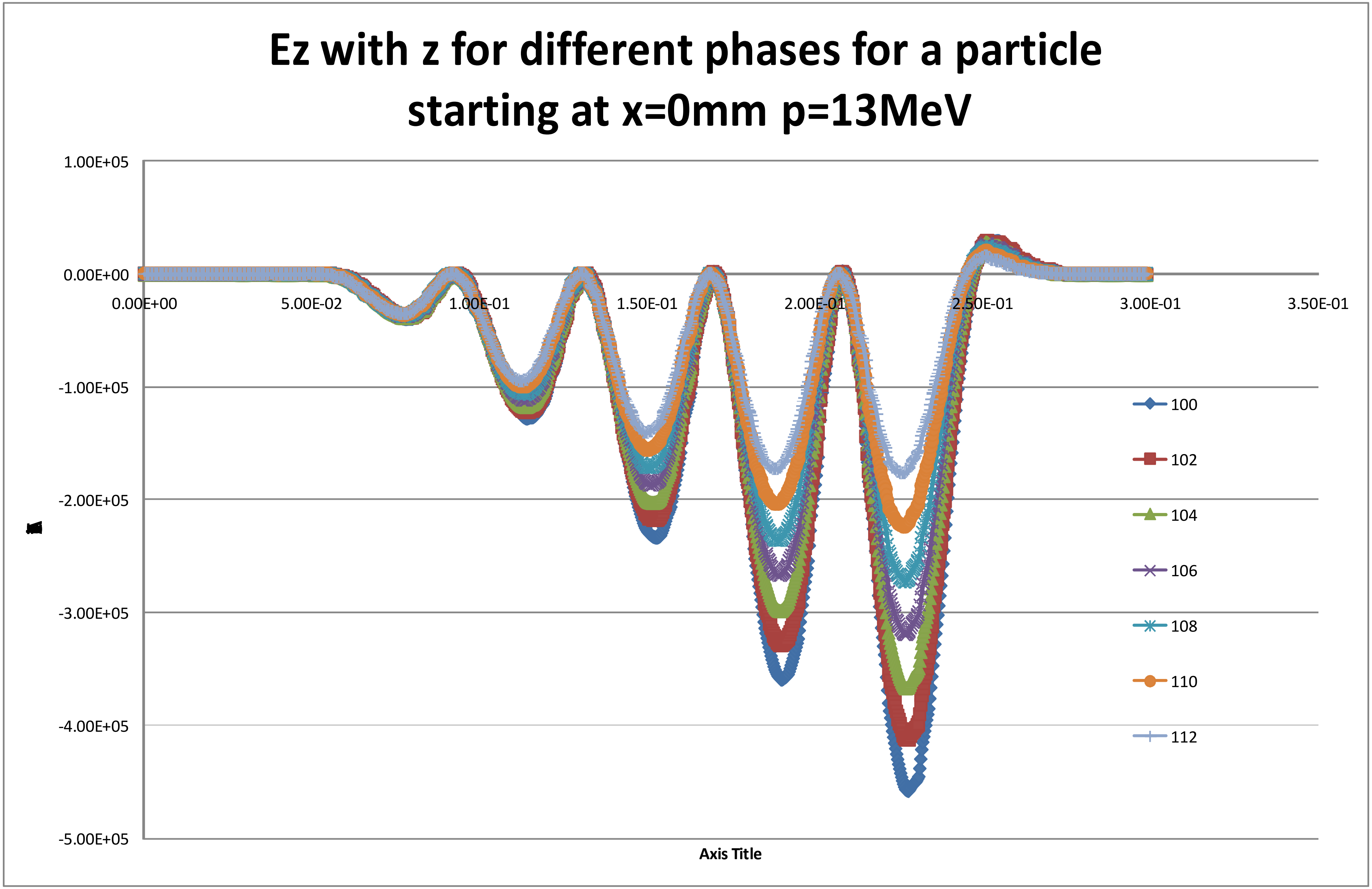



Ez seen by the probe particles with z
8.00E+05
6.00E+05
4.00E+05
2.00E+05
0.00E+00
-2.00E+05
-4.00E+05
-6.00E+05
0.00E+00
5.00E+01
1.00E+02
1.50E+02
2.00E+02
2.50E+02
3.00E+02
3.50E+02
z in mm
dp/p offset
z offset
y' offset
y offset
x' offset
x offset
Ref Particle



X of the probe particles with z
4.00E+00
3.50E+00
3.00E+00
2.50E+00
2.00E+00
1.50E+00
1.00E+00
5.00E-01
0.00E+00
-5.00E-01
in
0.00E+00
5.00E+01
1.00E+02
1.50E+02
2.00E+02
2.50E+02
3.00E+02
3.50E+02
z in mm
dp/p offset
z offset
y' offset
y offset
x' offset
x offset
Ref Particle


Ez seen by the probe particles with z
2.00E+05
0.00E+00
-2.00E+05
-4.00E+05
-6.00E+05
-8.00E+05
-1.00E+06
-1.20E+06
-1.40E+06
0.00E+00
5.00E+01
1.00E+02
1.50E+02
2.00E+02
2.50E+02
3.00E+02
3.50E+02
z in mm
dp/p offset
z offset
y' offset
y offset
x' offset
x offset
Ref Particle


Ez with z for different phases for a particle starting at x=0mm p=13MeV
1.00E+05
0.00E+00
-1.00E+05
-2.00E+05
-3.00E+05
-4.00E+05
-5.00E+05
0.00E+00
5.00E-02
1.00E-01
1.50E-01
2.00E-01
2.50E-01
3.00E-01
3.50E-01
100
102
104
106
108
110
112
Axis Title