Vectors and Coordinates

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1 System of coordinates

1.1 The example of Polar vs Cartesian coordinates

Polar to Cartesian:

$$x = r\cos\theta \tag{1}$$
$$y = r\sin\theta \tag{2}$$

Cartesian coordinates to polar coordinates:

$$r = \sqrt{y^2 + x^2} \tag{3}$$

$$\theta = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0 \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0 \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0 \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0 \\ 0 & \text{if } x = 0 \text{ and } y = 0 \end{cases} \tag{4}$$

This is a change of coordinate not a change of basis nor a change of inertial frame, and there is no bijection from one system of coordinate to the other because each point of the plan has an infinite number of polar coordinates but only one expressian in cartesian coordinates.

1.2 System of coordinates

A system of coordinates is used to establish a unique relation between an ordered tuple and a point P in space:

$$(x_1, ..., x_n) \rightarrow P \tag{5}$$

Note:

- the points in space exists independently of the coordinate system.

- this relation is not a bijection: As the example of the polar coordinate system because each point in space can be expressed by an infinity of coordinates.

- No operation applied on coordinates is introduced here, it requires a specific formalism (e.g. vectorial space). Using the polar coordinates again, in general[1]:

$$(r_1, \theta_1) \quad "+" \quad (r_2, \theta_2) \quad "\neq" \quad (r_1 + r_2, \theta_1 + \theta_2) \tag{6}$$

In a finite vectorial space with a given basis, coefficients resulting from the projection of the vectors on this basis can be used as coordinates.

1.3 Transformation of coordinates

A transformation of coordinates is a conversion from one system to another, to describe the same space. There is no reason for this transformation to be a bijection.

In the case of a vectorial space, expressing coordinates from one basis to another is a linear transformation that can be represented as a matrix.

1.4 Application

$$\tan\theta = \frac{T_y}{T_x} \sim \frac{-M_x}{M_y} \sim \frac{M_{\mathrm{NS}}}{M_{\mathrm{EW}}} \tag{7}$$

The wind angle can now we determined via a transformation $\mathcal{T}$, which converts angles counter-clockwise to clockwise, and that shifts the angles of $\pi/2$ for the origin to be along ON. By doing this, one will notice that the angles are not exactly corresponding to the one from the database. To get a better match, one has to add 13°to the resulting angles. This means that the strain gauges have been placed so that the axis EW is actually along the direction 283°. Eventually the wind direction is obtained by:

$$\theta_w = \mathcal{T}\left(\mathrm{atan}_2\left(\frac{M_{\mathrm{NS}}}{M_{\mathrm{EW}}}\right)\right) \cdot \frac{180°}{\pi} + 13° \tag{8}$$

Note one the convention Another way of proceeding consists in using the convention on the right side of Figure 1. This has the advantage to give directly the angles in the clockwise direction with respect to the NS axis.

[1] Find as an exercise the condition when it is actually an equality

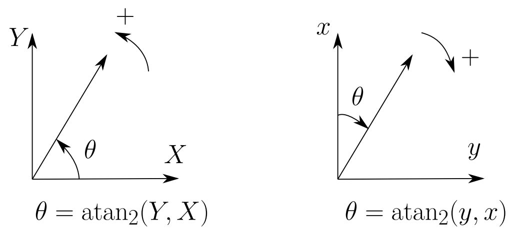

Figure 1: Angles convention, and corresponding inverse function

2 Definitions

A vector is an object linking two points. It has a length and a direction. Its nature is independent of the system of coordinates and frame. Nevertheless, it will have different expression in different coordinate systems and different frames. s

2.1 Change of coordinate system

3 Vector derivation:

$$\left.\frac{d\underline{r}}{dt}\right|_{R} = \left.\frac{d\underline{X}}{dt}\right|_{R'} + \underline{\omega}_{\left(\frac{R'}{R}\right)} \times \underline{r} \tag{9}$$